MULTIMIX 2026

Program overview

Thursday - September 3

13:30-14:00	Coffee/Tea
14:00-14:10	Opening
14:10-15:40	Talks
15:40-16:10	Coffee/Tea
16:10-17:40	Talks
17:40-19:30	Break
19:30-	Dinner

Friday - September 4

09:15-10:45	Talks
10:45-11:15	Coffee/Tea
11:15-12:45	Talks
12:45-13:00	Closing

Location

Johannes Gutenberg-University Mainz
Staudingerweg 7, 55128 Mainz, Germany
`https://maps.app.goo.gl/TYr9uJopsXt4f5NH7`

Room: Minkowski room, number 05 119

Dinner

Alex Mainz
Gutenbergplatz 14, 55116 Mainz, Germany
`https://maps.app.goo.gl/qDPJLvHPG377DZD49`

Schedule

Thursday - September 3

13:30-14:00	**Coffee/Tea**
14:00-14:10	**Opening**
14:10-14:40 Helmut Abels	Local well-posedness for a diffuse-interface mixture model for an incompressible two-phase flow
14:40-15:10 Marc Hirschvogel	Multiphase fluid-structure interaction: Preconditioners and stabilization schemes for the Cahn-Hilliard-Navier-Stokes equations in a moving reference frame
15:10-15:40 Makrand Khanwale	Unravelling the complexities of breakup dynamics in turbulent multiphase flows through the Cahn-Hilliard Navier-Stokes phase-field model
15:40-16:10	**Coffee/Tea**
16:10-16:40 Mathis Plapp	Phase-field simulations of viscous fingering in shear-thinning fluids
16:40-17:10 Karel Tuma	From total spreading to realistic Ti–Mo microstructures: phase-field modelling of the $\beta \rightarrow \omega$ transformation
17:10-17:40 Dennis Trautwein	Energy-stable and convergent numerical methods for viscoelastic fluid models
17:40-19:30	**Break**
19:30-	**Dinner**

Friday - September 4

09:15-09:45 Ansgar Jüngel	Multiphase multicomponent cross-diffusion models for tissue structures
09:45-10:15 Jonathan Stollberg	A Lagrangian mixture theory for growing multiphase tissues with application to avascular tumors
10:15-10:45 Andrea Thomann	Entropy stable semi-implicit scheme for the Euler equations

10:45-11:15	**Coffee/Tea**

11:15-11:45 Dieter Bothe	Towards global existence for a class of Maxwell–Stefan diffusion systems
11:45-12:15 Marco ten Eikelder	Thermodynamic closure of the N-phase Navier–Stokes–Cahn–Hilliard mixture model
12:15-12:45 Aaron Brunk	Structure-preserving approximations of N-phase flow with unmatched densities

12:45-13:00	**Closing**

Organizers

Aaron Brunk, Marco ten Eikelder

Participants

Speakers	Participants
1. Helmut Abels	1. Dennis Höhn
2. Dieter Bothe	2. Marvin Schmidt
3. Aaron Brunk	3. Olga Schewe
4. Marco ten Eikelder	4. Samuel Medina Narvaez
5. Marc Hirschvogel	
6. Ansgar Jüngel	
7. Makrand Khanwale	
8. Mathis Plapp	
9. Jonathan Stollberg	
10. Andrea Thomann	
11. Dennis Trautwein	
12. Karel Tuma	

Abstracts

Helmut Abels

Local well-posedness for a diffuse-interface mixture model for an incompressible two-phase flow

We discuss local in time well-posedness of a diffuse interface model for the two-phase flow of viscous incompressible fluids proposed by ten Eikelder, van der Zee and Schillinger. In contrast to the usual models of Navier–Stokes/Cahn–Hilliard (or Allen–Cahn)-type the model is derived from full mixture theory, taking the conservation of linear momentum for the both components into account. First we prove a result on maximal L^2-regularity for the linearized system. Then a fixed-point argument is used to show the strong well-posedness locally in time. This is a joint-work with Harald Garcke and Julia Wittmann.

Dieter Bothe

Towards global existence for a class of Maxwell–Stefan diffusion systems

Maxwell-Stefan diffusion describes multicomponent transport through pairwise friction and leads to singular, strongly coupled cross-diffusion systems.

The talk presents a constitutive class for which the constrained diffusion operator admits an unexpectedly explicit structure. After recalling the modelling background and the analytical status of general Maxwell–Stefan systems, the talk focuses on spectral coordinates for the composition variables and on a complementary entropy-symmetric formulation. These structures yield invariant-region properties and substantially stronger a priori control than is available in the general case. This permits significant progress towards global strong solvability for strictly positive initial compositions without smallness assumption.

Aaron Brunk

Structure-preserving approximations of N-phase flow with unmatched densities

This work addresses the modeling and numerical treatment of N-phase mixtures with unmatched densities, formulated using the mass-averaged velocity. The resulting system consists of partial differential equations governing the evolution of volume fractions coupled with a quasi-incompressible Navier–Stokes equation for the flow, subject to a saturation constraint. We focus on reformulating the continuous problem to provide a suitable foundation for structure-preserving approximations using standard finite elements and time integration schemes.

Our framework ensures consistency with the underlying thermodynamic and geometric structures, enabling stable and physically meaningful simulations. The proposed reformulations simplify the governing equations while preserving essential invariants such as mass conservation and energy dissipation. To illustrate the potential of these methods, we present preliminary numerical tests demonstrating robustness and accuracy in representative scenarios.

Marco ten Eikelder

Thermodynamic closure of the N-phase Navier–Stokes–Cahn–Hilliard mixture model

We develop a reduction-consistent thermodynamic closure for N-phase Navier–Stokes–Cahn–Hilliard mixture models and a practical calibration procedure for prescribed pairwise surface tensions. Working within a first-gradient diffuse-interface framework, we formulate a set of structural assumptions under which a capillary-force reduction constraint is satisfied: when two components are physically identical and merged, the N-phase formulation reduces to the $(N-1)$-phase formulation. Under these assumptions we derive the admissible free-energy structure and the admissible mobility tensor. Building on the resulting parametrization, we reconstruct model coefficients from target pairwise surface tensions. We validate our findings through numerical experiments. This is joint work with Aaron Brunk.

Marc Hirschvogel

Multiphase fluid-structure interaction: Preconditioners and stabilization schemes for the Cahn-Hilliard-Navier-Stokes equations in a moving reference frame

Authors: Marc Hirschvogel, John Paul Gerakis, Erik Esche

Multiphase fluid flow and its interaction with deformable structures are of significant interest across many engineering disciplines. From a computational perspective, however, the modeling of these phenomena remains challenging, and comparatively little attention has been devoted to robust numerical schemes for fluid-structure interaction involving multiple fluid phases [2,3].

Here, we present a monolithic computational approach based on a mass-averaged velocity formulation of the Cahn-Hilliard-Navier-Stokes (CH-NS) equations [1] in a moving (ALE) reference frame, coupled to finite-strain elastodynamics. The proposed formulation provides robustness for strongly coupled and highly dynamic problems. Particular emphasis is placed on efficient preconditioners for the coupled system, for which we propose a forward Block Gauss-Seidel scheme for the solid-grid-multifluid problem, with special attention devoted to the CH-NS block. In addition, we investigate stabilization schemes suitable for equal-order P1-P1 interpolation of velocity and pressure, aiming to achieve robust and stable discretizations while maintaining accuracy.

The proposed approach is demonstrated for the interaction of a collapsing water column with a deformable obstacle (dam-break) surrounded by air, illustrating robust performance for high density-ratio multiphase flows coupled to dynamic structural deformation.

1 Brunk, A. and ten Eikelder, M. F. P. (2026). A simple, fully-discrete, unconditionally energy-stable method for the two-phase Navier-Stokes Cahn-Hilliard model with arbitrary density ratios. Journal of Computational Physics, 548:114558. https://doi.org/10.1016/j.jcp.2025.114558

2 Valizadeh, N., Zhuang, X., and Rabczuk, T. (2025). A monolithic finite element method for phase-field modeling of fully Eulerian fluid-structure interaction. Computer Methods in Applied Mechanics and Engineerings, 435:117618. https://doi.org/10.1016/j.cma.2024.117618

3 Zheng, X. and Karniadakis, G. E. (2016). A phase-field/ALE method for simulating fluid-structure interactions in two-phase flow. Computer Methods in Applied Mechanics and Engineering, 309:19–40. https://doi.org/10.1016/j.cma.2016.04.035

Ansgar Jüngel

Multiphase multicomponent cross-diffusion models for tissue structures

Multiphase models provide a continuum framework to investigate interactions between different components of a cellular fluid. Assuming force balances for the interphase pressures and viscous drag forces, the evolution of the volume fractions of the mixture phases is governed by nonlinear diffusion systems. In this talk, we formally derive a novel class of volume-filling cross-diffusion equations is formally derived and investigate the energy/entropy structure. Using entropy methods, we establish the existence of bounded weak solutions, analyze their long-time behavior, and prove weak–strong uniqueness for a regularized system. Interestingly, for nonconstant Maxwell–Stefan-type drag coefficients, the diffusion matrix remains positively stable, whereas the mobility matrix associated with the Boltzmann entropy may possess negative eigenvalues.

Makrand Khanwale

Unravelling the complexities of breakup dynamics in turbulent multiphase flows through the Cahn-Hilliard Navier-Stokes phase-field model

One of the long-standing challenges in multiphase flows that involve deforming interfaces is predictive modelling of the process of interface breaking apart. This is a multiscale phenomenon that involves effects at the molecular scale. Understanding and predicting this process is crucial for many manmade and natural phenomena, such as atomization, disease spread, energy conversion, inkjet printing, biological flows, and wave breaking in the ocean. "High-fidelity" numerical simulations can be valuable in understanding such complex multiscale phenomena. In this talk, I will discuss the formulation and numerical discretisation of phase-field models for simulating multiphase flows involving complex breakup phenomena. I will detail advances in numerical analysis to maintain the stability and accuracy of discretisation, and computational strategies that enable unprecedented scale resolution while keeping computational costs low, including the deployment of the discretisation on a massively parallel adaptive meshing framework. I will use test problems of progressively increasing complexity in both low and high Reynolds number regimes to demonstrate the effects of modelling and computational approaches. I will then present simulations and analysis of the canonical pulsed primary jet atomization problem that demonstrates the capabilities of the computational framework. In this case, I will discuss the intricacies of atomization dynamics, which involve a complex cascade of instabilities and breakup mechanisms ranging from sheet rupture to filament formation, culminating in droplet pinch-off. To quantitatively assess the effect of our approach on breakup statistics, the droplet size distribution will be analysed and put in context with other current state-of-the-art modelling approaches.

Mathis Plapp

Phase-field simulations of viscous fingering in shear-thinning fluids

Sebastien Nguyen, Hervé Henry, Mathis Plapp

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Viscous fingering occurs when a low-viscosity fluid displaces a high-viscosity fluid in a Hele-Shaw cell or in a porous medium. For Newtonian fluids in a Hele-Shaw cell, a single finger typically develops. The Saffman-Taylor problem for the selection of the finger width has been instrumental for the understanding of the physics of branching patterns. Much less is known for non-Newtonian fluids, in which the dependence of the fluid properties on local flow parameters modifies the pressure distribution around the finger. We have developed a phase-field model for viscous fingering, which extends a previous model for Newtonian fluids to a wide range of fluids with a shear-dependent viscosity. The model is applied to simulate viscous fingering in shear-thinning fluids and found to capture the complete crossover from the Newtonian regime at low-shear rate to the strongly shear-thinning regime. The width selection of a single steady-state finger is studied in detail for a two-plateau shear-thinning law (Carreau's law) in both its weakly and strongly shear-thinning limits, and the results are related to previous analyses. For power-law – Ostwald–deWaele – fluids in the strongly shear-thinning regime, good agreement with experimental data from the literature is obtained.

Jonathan Stollberg

A Lagrangian mixture theory for growing multiphase tissues with application to avascular tumors

Biological growth involves the coupled evolution of deformation, transport, and mass exchange between multiple interacting constituents. We present a thermodynamically consistent continuum framework for growing mixtures with an arbitrary number of solid and fluid phases. The formulation combines constituent-wise balance laws with the multiplicative decomposition of the solid-skeleton deformation gradient. An energy–dissipation principle is used to identify the thermodynamic driving forces associated with elastic deformation, growth, transport, and inter-phase mass exchange, thereby providing a systematic basis for constructing admissible constitutive relations.

The framework is specialized to avascular tumor growth by representing the tissue as a fully saturated mixture of different cell populations and interstitial fluid. Nutrient-dependent proliferation and phenotype changes, together with mechanical interactions between the constituents, are incorporated through suitable free-energy and mobility terms. Numerical benchmark problems illustrate the resulting coupling between nutrient availability, cellular composition, and growth-induced deformation and demonstrate the applicability of the formulation to multiphase biological growth processes. This is joint work with Marco ten Eikelder and Dominik Schillinger.

Andrea Thomann

Entropy stable semi-implicit scheme for the Euler equations

In this work we propose an entropy stable semi-implicit scheme for the Euler equations applicable in all Mach number regimes. It is characterized by stability under large time steps oriented towards the convective processes while acoustic waves are integrated implicitly and thus do not contribute to the CFL criterion.
The scheme is characterized by advancing the entropy directly instead of the total energy which is conserved as a consequence by a correction procedure. Due to the hyperbolic thermodynamical compatibility of the Euler equations, the entropy inequality is obtained and discretized which allows to advance the entropy as primary variable. The body of the numerical scheme is given by an implicit-explicit scheme inspired by the work of Casulli.
The properties of the scheme are illustrated by one and two-dimensional test cases. If time permits an outlook towards the application to two-phase flows will be given.
This is joint work with Sara Rinaldi (Univ. Trento, Italy), Gabriella Puppo (La Sapienza Univ. Roma, Italy) and Michael Dumbser (Univ. Trento, Italy).

Dennis Trautwein

Energy-stable and convergent numerical methods for viscoelastic fluid models

Viscoelastic fluid models couple the incompressible Navier-Stokes equations with nonlinear evolution equations for the elastic stress tensor (such as Oldroyd-B or Giesekus). Despite their importance, developing numerical schemes that remain energy-stable and convergent under large elastic deformations remains a significant challenge. In this presentation, we review prominent numerical strategies from the literature and address their properties like energy-stability and rigorous convergence. We analyze key techniques such as the log-conformation representation, discrete chain-rule approximations, and structure-preserving methods utilizing auxiliary variables. While the rigorous convergence of such methods is generally unknown, we present a novel (subsequence) convegence result for the Giesekus model, notably without artificial regularization in the limit system. Finally, we demonstrate the robustness of these methods through numerical tests at high Weissenberg numbers, where conventional schemes typically lose positivity of the conformation tensor.

Karel Tuma

From total spreading to realistic Ti–Mo microstructures: phase-field modelling of the $\beta \rightarrow \omega$ transformation

Motivated by the athermal $\beta - \omega$ transformation in Ti–Mo alloys, we develop a multiwell phase-field model for the resulting multivariant microstructure. Different ω variants are prevented from forming direct interfaces and remain separated by thin layers of the parent β phase. The model includes anisotropic elasticity, phase-dependent stiffness tensors, a chemical driving force based on the local molybdenum concentration and temperature, and Langevin noise enabling nucleation. The molybdenum distribution is obtained from atomistic preprocessing with realistic short-range order. Three-dimensional simulations of cooling in Ti-15Mo reproduce preferential nucleation in chemically favorable regions, growth of four ω variants, and their separation by the β phase. The simulated microstructure evolution is compared with experimental data.